

Linear Relationships

OBJECTIVE

Given any one of the four representations of a linear relationship (words, data, equation, graph), students will be able to generate the other three, understanding the role of slope and y-intercept in each.

LESSON PLAN

This lesson has the capacity to be a stand-alone introductory unit on linear equations. By the time the students have completed all phases of the lesson, they should be able to flow from one representation to another. Interestingly, the three representations that are traditionally taught (data, equation and graph) can be generated, one from another, through an algorithm. The fourth (words) requires a higher level of thinking on the part of the student. This critical thinking is needed in both directions, for generating the words from any of the other three representations, as well as for figuring the first representation from words. Therefore, it is important to guide the students artfully through this process.

This lesson stemmed from MPJ's *Rising Water* lesson, by Jennifer Sawyer. It is suggested to use that lesson as an introduction to the four representations, then to tackle each of them separately. For each representation, three problems are given. The first may be used as an example, the second as guided practice, and the third as independent practice. The teacher may also decide to give all three as an independent, problem-solving activity that is then publicly debriefed in class. Either method allows for two representations to be studied within one class period, so all can be covered in two days with a third day dedicated to the assessment. The Standard Form lesson is optional and would take an additional class period. The lesson is in a similar format to the previous 4 x 4 lessons, however, the scenario will focus on the coefficients of the variables rather than the slope and y-intercept. The data for the Standard Form is limited to x- & y- intercepts.

Notes on Domain & Range and on Discrete vs. Continuous

Once a scenario is applied to a graph, there is often a demand to restrict the domain and range, and to determine if the graph should be drawn as continuous or discrete. For example, "Fred has no friends, then makes a new friend every day," compels an experienced mathematician to draw a one-quadrant graph instead of four-quadrant, and to draw a dotted line rather than a solid one. That line would not have any arrows at the end because Fred will not make friends forever. In order to keep this lesson focused on the meaning of slope and y-intercept across the four representations, the two tasks of defining the domain & range and determining whether the scenario is discrete or continuous is suspended for this lesson.

Concepts

Four representations of linear relationships (words, data, equation and graph)

Time: 2-4 hours.

The Standard Form is an ancillary component of the lesson.

Materials

Student Handouts

Preparation

Conducting the **Rising Water** lesson first serves as an overview of the four representations and establishes the structure for this lesson.

LESSON PLAN

4 × 4 (continued)

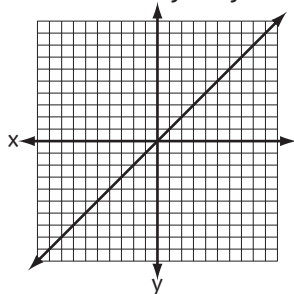
However, it should be mentioned to the students that the equations and graphs are generalizations of ALL the potential scenarios. Data is traditionally treated in a similar fashion. A table offers only a minute subset of ALL the infinite points offered by the equation and the graph. Therefore, in the same way that their data table represents only part of the line, the scenario only represents part of the line and might only represent part of the data table. The only requirement here is that the slope and y-intercept correspond for all four representations. *NOTE: For the answer keys, the data and scenarios generated by the students are open ended, so only samples only are offered here. Students responses may vary.*

REPRESENTATION #1: Words/Scenario

The focus of each scenario is to illuminate in the students' minds that the slope is some kind of rate of change and that the y-intercept is a "starting point" when the value of x is 0. This phase of the lesson also stresses that slope and y-intercept can be used to generate the graph and the table. Each scenario emphasizes a different concept of these two elements. Fred's story shows that even when we don't "see" any numbers when the slope is one and the y-intercept is zero, those values are still there. Jamie demonstrates what a negative y-intercept looks like. Ali presents a slope that is both negative and not an integer. The top of the data table is purposely left blank so the students must establish the variables.

1) $y = x$

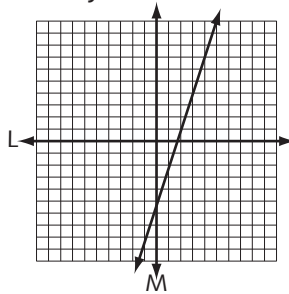
Fred moves into town with no friends, and makes one new friend every day.



x	y
0	0
1	1
2	2
3	3
4	5

2) $M = 3L - 5$

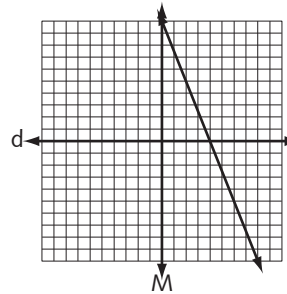
Jamie spent \$5 to start his lemonade stand and made \$3 every hour.



L	M
-2	-11
-1	-8
0	-5
1	-2
2	1

3) $M = -\frac{5}{2}d + 10$

Ali has \$10 and he spends \$5 every two days.



d	M
0	10
2	5
4	0
6	-5
8	-10

REPRESENTATION #2: Data

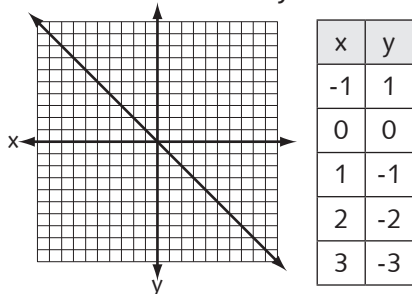
Here the student can plot points to generate the graph. The opportunity offered by this phase of the lesson is the idea of the y-intercept having an x-value of zero. The students can actually see it in the table. From there, the teacher can demonstrate how the slope can be determined by the change in y in the table, over the change in x , and how that applies to the graph. There is a blank entry at the bottom of the data table, to invite the students to continue the pattern of numbers in each side of the data table, further emphasizing the slope. From those two components of the linear relationship, the equation and the scenario can be written. In the key below, the scenario is offered as a sample, because there are an infinite number of scenarios for a given slope and y-intercept.

LESSON PLAN

4 × 4 (continued)

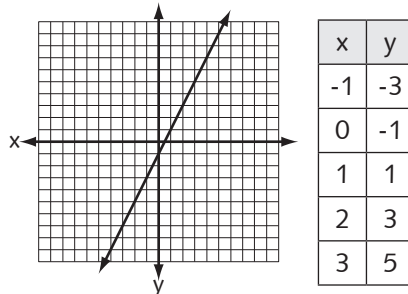
4) $y = -x$

A scuba diver starts on the surface of the water, and then dives 1 meter every second.



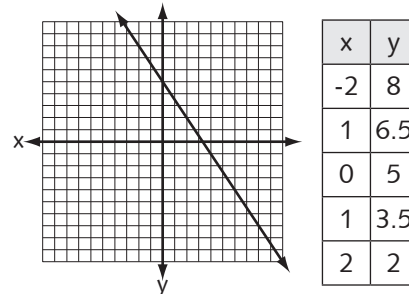
5) $y = 2x - 1$

Jackson owes a dollar, but wins \$2 on every bet.



6) $y = -\frac{3}{2}x + 5$

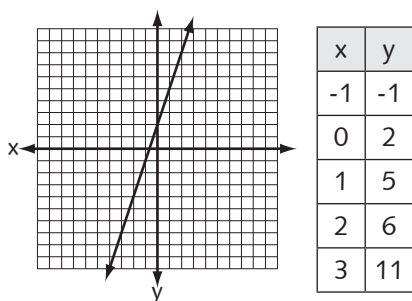
Trent has \$5, and charges \$3 to his credit card for every 2 stores.

**REPRESENTATION #3: Equation**

This is probably the most traditional phase of the lesson: having students create a table of data and a graph from an equation. The students are still to create one of a number of scenarios from the slope and y-intercept. It is suggested that the students be encouraged to choose at least one negative x-value in the data table, as well as multiples of the slope's denominator.

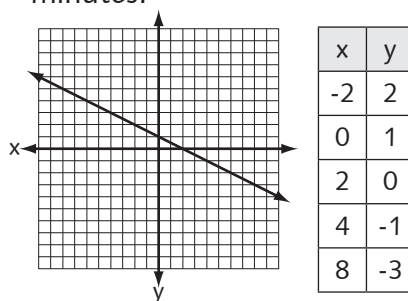
7) $y = 3x + 2$

Troy has 2 zits, and he gets 3 more zits every morning.



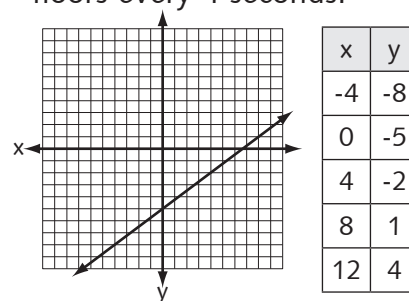
8) $y = -\frac{1}{2}x + 1$

At sunset, the temperature is 1° C and drops 1° every 2 minutes.



9) $y = \frac{3}{4}x - 5$

Sue enters the elevator 5 floors underground, and it rises 3 floors every 4 seconds.

**REPRESENTATION #4: Graph**

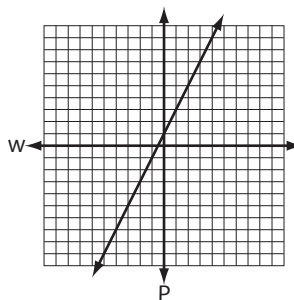
By now, the students should be able to easily extract data points from the graph, determine its slope and y-intercept, and thus, write an equation and scenario. Remind students that their scenario might only represent a subset of the data table. For example, in the sample for number #11 below, we could conceivably have negative time (Vicki may have had more donuts a few minutes ago), but she will never have negative donuts. Also, encourage the students to use variables that match the scenario, rather than simply x and y, in their table and graph. This will force them to wrestle with defining independent versus dependent variable.

LESSON PLAN

4 × 4 (continued)

10) $P = 2w + 1$

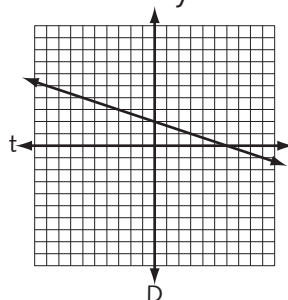
Wally has 1 poster and hangs 2 new ones every week.



w	P
-1	-1
0	1
1	3
2	5
3	7

11) $D = -\frac{1}{3}t + 2$

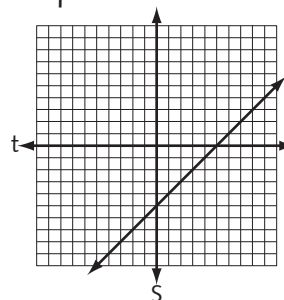
Vicky has 2 donuts and is devouring them at a rate of one every 3 minutes.



t	D
-3	3
0	2
3	1
6	0
9	-1

12) $S = t - 5$

Casper starts 5 steps below the landing and climbs 1 step per second.



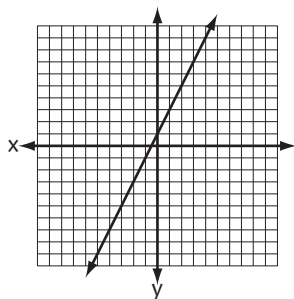
t	S
0	-5
1	-4
2	-3
3	-2
4	-1

ASSESSMENT

This assessment is the ultimate test for the unit. Can the students, given any one of the four representations of a linear relationship, generate the other three?

1) $y = 2x + 1$

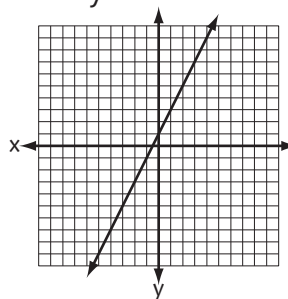
Lucy has one bracelet and buys 2 more each month.



x	y
-2	-3
0	1
2	5
4	9

2) $y = 2x + 1$

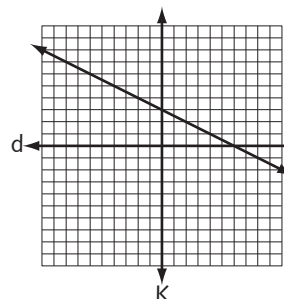
Sweeny has already emptied 1 trash can and gets paid \$2 for every other can he empties.



x	y
-1	-1
0	1
1	3
2	5

3) $K = -\frac{1}{2}d + 3$

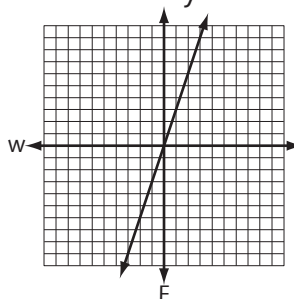
Bri has 3 keys and loses a key every 2 days.



d	K
-2	4
0	3
2	2
4	1

4) $F = 3w$

Suzie enters her school with no friends, and makes 3 new friends every week.



w	F
0	0
1	3
2	6
3	9

4 × 4 (continued)

OPTIONAL ACTIVITY: Standard Form

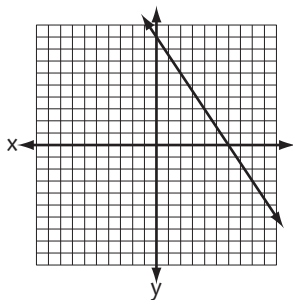
This optional activity/assessment is to be used after students have been introduced to the standard form of a line and have had experience with computing the intercepts. The emphasis of the context here moves away from the meaning of slope and y-intercept to the meaning of the coefficient of the variables and of the constant. Students will need extensive familiarity with standard form word problems in order to be able to generate scenarios like the samples below.

1) $3x + 2y = 18$

y-intercept: (0,9)

x-intercept: (6,0)

Dale makes \$3 for every lawn that he mows and \$2 for every hedge that he trims. He made a total of \$18.



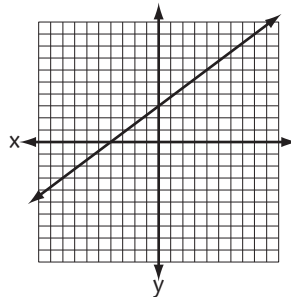
x	y
0	9
6	0

2) $3x - 4y = -12$

y-intercept: (0,3)

x-intercept: (-4,0)

Dale makes \$3 for every lawn he mowed and pays \$4 for every gallon of gas. He lost \$12 on his landscaping business this week.



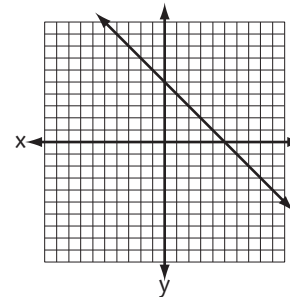
x	y
0	3
-4	0

3) $x + y = 5$

y-intercept: (0,5)

x-intercept: (5,0)

Two numbers always have a sum of 5.



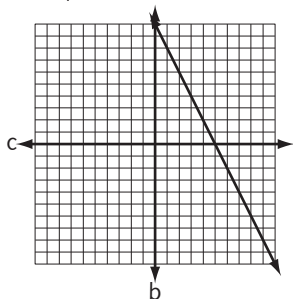
x	y
0	5
5	0

4) $2c + b = 10$

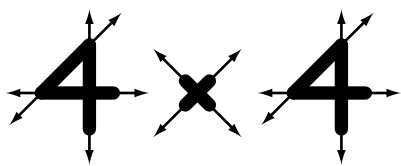
y-intercept: (0,10)

x-intercept: (5,)

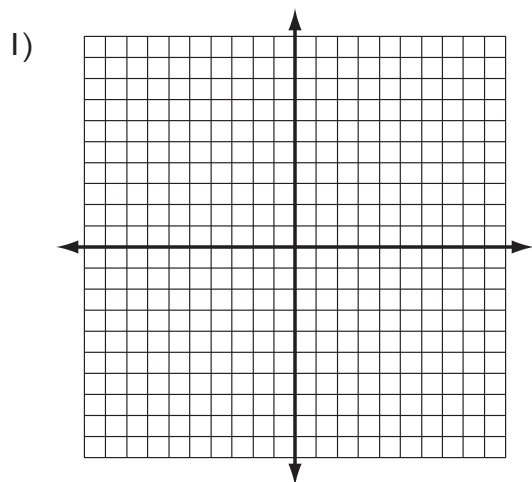
Jasmine gets paid \$2 for every cupcake and \$1 for every brownie. She receives a total of \$10.



c	b
0	10
5	0

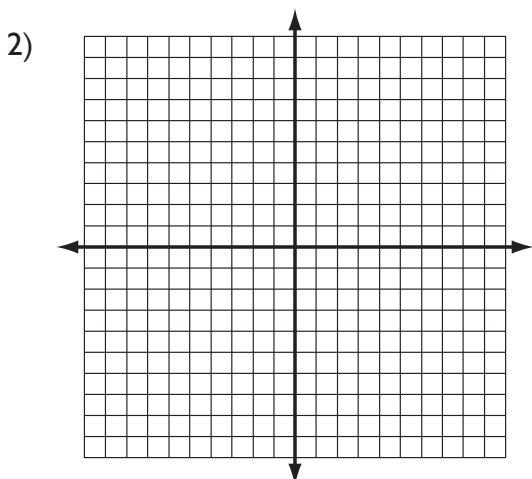


From Scenarios/Words



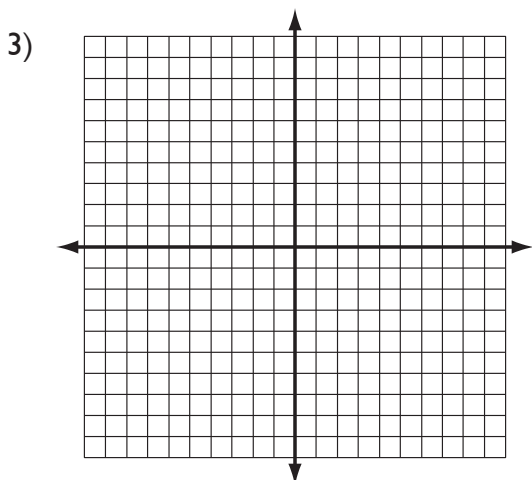
Equation:

Scenario: **Fred moves into town with no friends, and makes one new friend every day.**



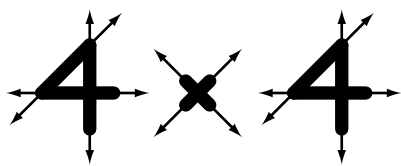
Equation:

Scenario: **Jamie spent \$5 to start his lemonade stand and made \$3 every hour.**



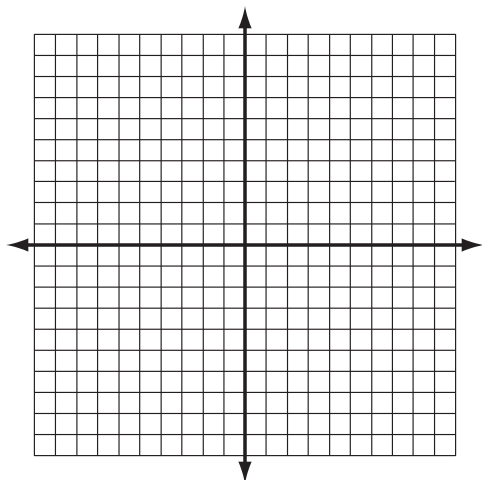
Equation:

Scenario: **Ali has \$10 and he spends \$5 every two days.**



From Data

4)

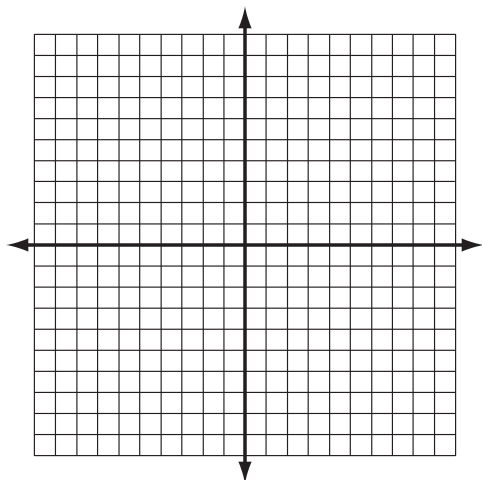


X	Y
-1	1
0	0
1	-1
2	-2

Equation:

Scenario:

5)

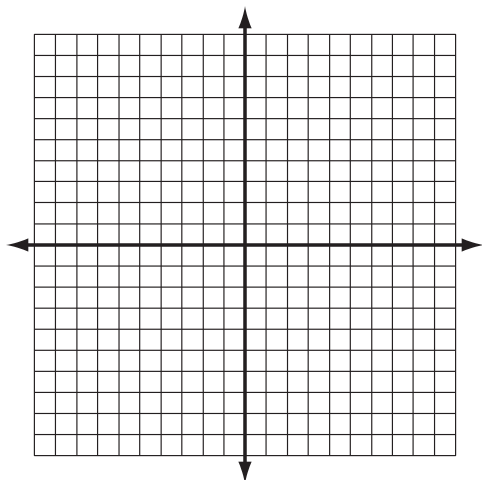


X	Y
-1	-3
0	-1
1	1
2	3

Equation:

Scenario:

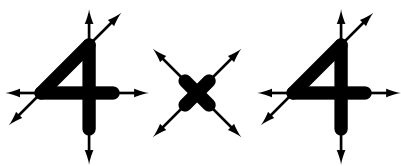
6)



X	Y
-2	8
-1	6.5
0	5
1	3.5
2	2

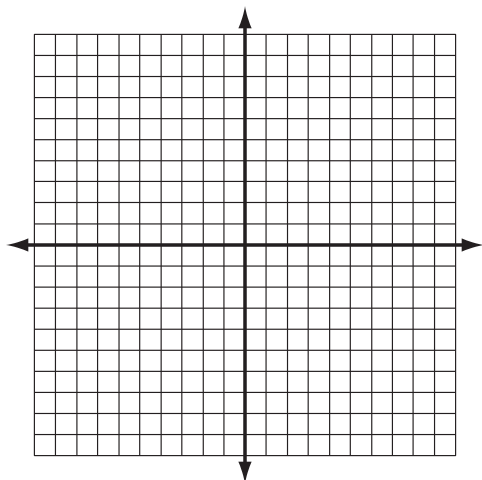
Equation:

Scenario:



From Equations

7)

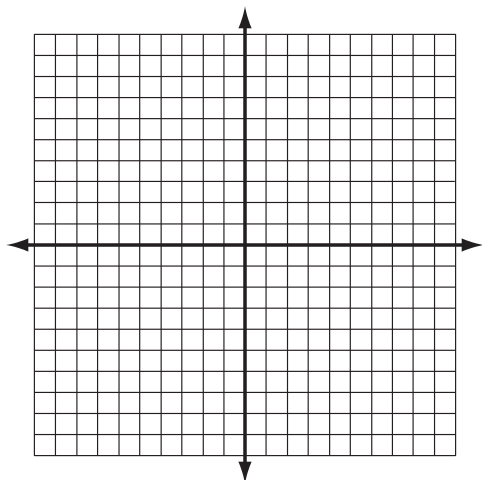


x	y

Equation: $y = 3x + 2$

Scenario:

8)

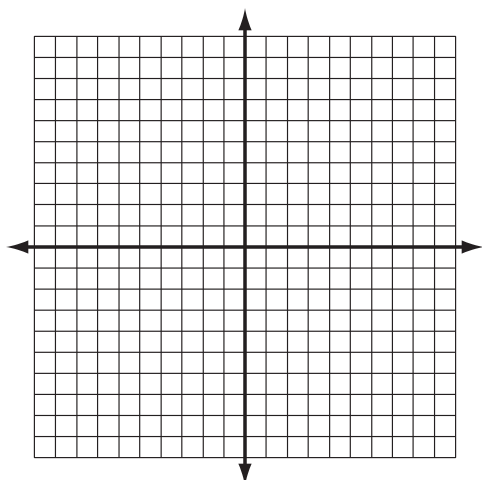


x	y

Equation: $y = -\frac{1}{2}x + 1$

Scenario:

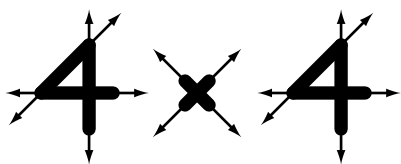
9)



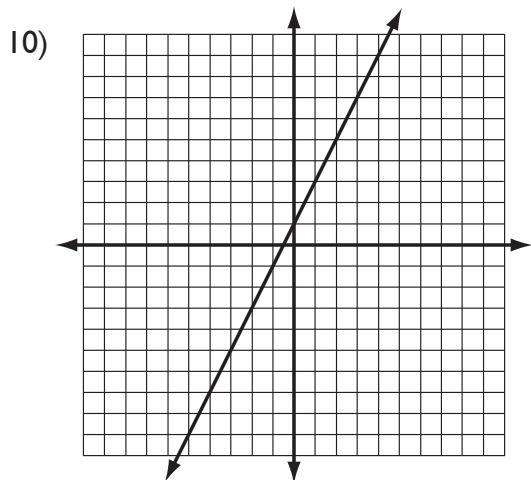
x	y

Equation: $y = \frac{3}{4}x - 5$

Scenario:

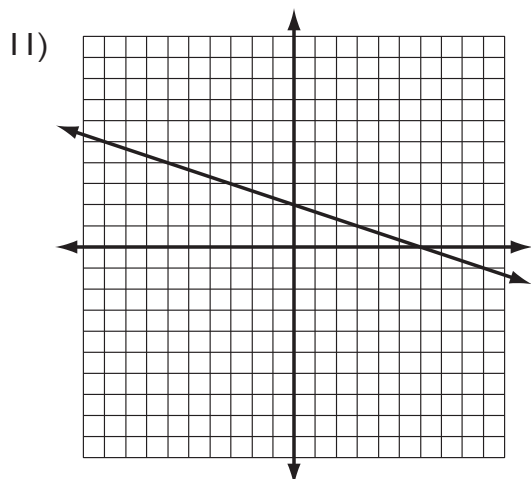


From Graphs



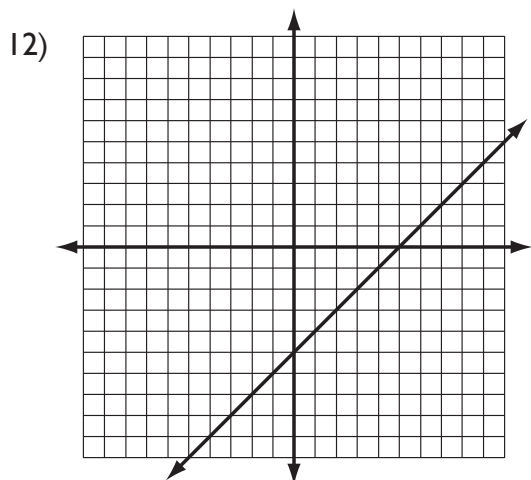
Equation:

Scenario:



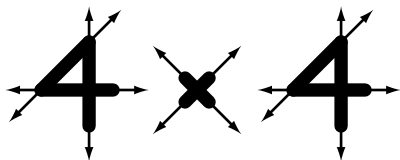
Equation:

Scenario:



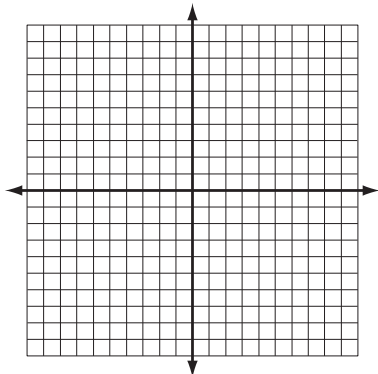
Equation:

Scenario:



Assessment

1)

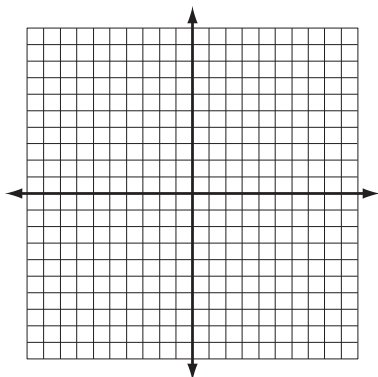


X	Y
-2	-3
0	1
2	5

Equation: _____

Scenario:

2)

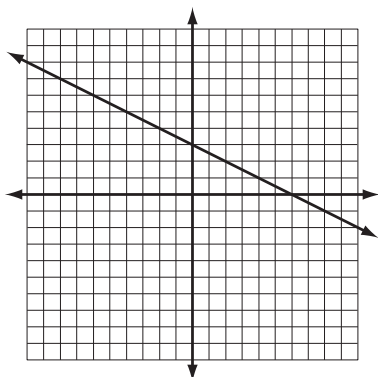


X	Y
-1	
0	
1	

Equation: $y = 2x + 1$

Scenario:

3)

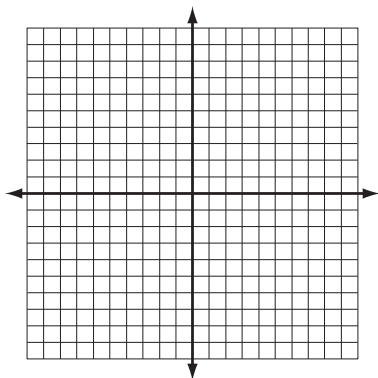


X	Y

Equation: _____

Scenario:

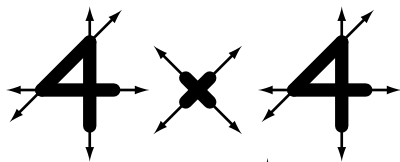
4)



X	Y

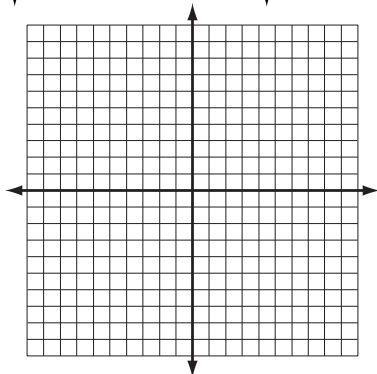
Equation: _____

Scenario: **Suzie enters her school with no friends, and makes 3 new friends every week.**



Standard Form

1)



X	Y
0	
	0

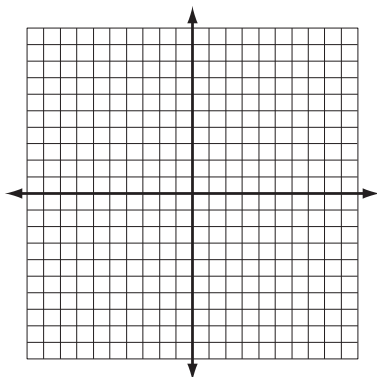
Equation: $3x + 2y = 18$

y-intercept: (,)

x-intercept: (,)

Scenario:

2)



X	Y
0	3
-4	0

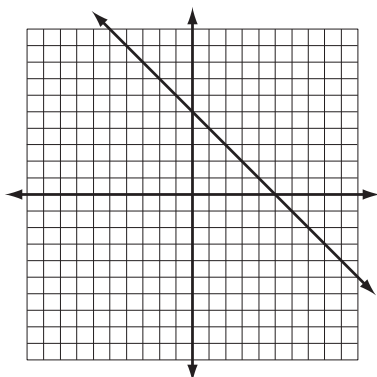
Equation: _____ = _____

y-intercept: (,)

x-intercept: (,)

Scenario:

3)



X	Y

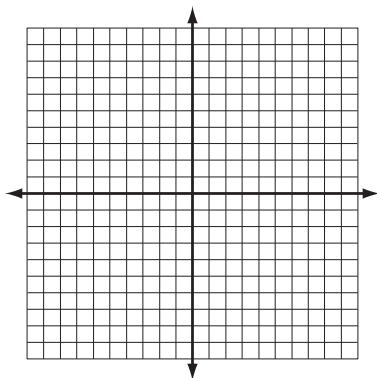
Equation: _____ = _____

y-intercept: (,)

x-intercept: (,)

Scenario:

4)



X	Y

Equation: _____ = _____

y-intercept: (,)

x-intercept: (,)

Scenario: Jasmine gets paid \$2 for every cupcake and \$1 for every brownie. She receives a total of \$10.